Sum rules for spin asymmetries

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Abstract

Starting from rotational invariance we derive sum rules for the single–spin asymmetries in inclusive production and binary processes. We also get sum rules for spin correlation parameters in elastic pp–scattering.

An important role of spin effects for analysis of hadron interaction dynamics is widely recognized nowadays. The space–time structure of the strong interactions provides a number of constraints for the spin observables (cf. [1]) and, as it will be shown further, allows us to get a useful sum rule for the single–spin asymmetries and spin correlation parameters.

Let us consider first single–spin asymmetry in hadron production

$$h_1 + h_2 \to h_3 + X,$$

where the beam or target hadron $h_{1,2}$ is transversely polarized. Let ξ stands for the set of variables related to the hadron h_3. The definition of asymmetry A_N is well known

$$A_N(s,\xi) = \left[\frac{d\sigma^{\uparrow}}{d\xi}(s,\xi) - \frac{d\sigma^{\downarrow}}{d\xi}(s,\xi)\right] / \left[\frac{d\sigma^{\uparrow}}{d\xi}(s,\xi) + \frac{d\sigma^{\downarrow}}{d\xi}(s,\xi)\right].$$

The equality of the integrated inclusive cross-sections follows from rotational invariance in a straightforward way, i.e.

$$\int \frac{d\sigma^{\uparrow}}{d\xi}(s,\xi)d\xi = \int \frac{d\sigma^{\downarrow}}{d\xi}(s,\xi)d\xi.$$

Then from the definition of A_N we have the following sum rule

$$\int A_N(s,\xi)\frac{d\sigma}{d\xi}(s,\xi)d\xi = 0, \tag{1}$$

where $d\sigma/d\xi$ is the unpolarized cross-section. Eq. (1) should be taken into account at the construction of models intended to explain the significant single–spin asymmetries observed in the inclusive processes. Similar sum rule (with replacement $A_N \to P$) takes place when the polarization of the final hadron h_3 can be measured (Λ–hyperon for example).

The above arguments can be applied to analyzing power in elastic and binary processes, e.g. from the equality

$$\sigma^{\uparrow}_{el}(s) = \sigma^{\downarrow}_{el}(s)$$

we should have

$$\int_{-s+4m^2}^{0} A(s,t)\frac{d\sigma}{dt}(s,t)dt = 0, \tag{2}$$

where $A(s,t)$ is the analyzing power in elastic scattering and $d\sigma/dt$ is the unpolarized cross–section for the elastic scattering of the particles with equal masses. Sum rule for the inelastic binary processes has the similar form with minor kinematical changes of the integration limits. *From Eq. (2) we arrive to conclusion*

that $A(s,t)$ should have sign–changing t-dependence since $d\sigma/dt$ is positive. This conclusion on the t–dependence is useful for the planning of the future experiments on the analyzing power measurements in elastic scattering at higher values of t [2]. It should be noted that change of sign of A in elastic pp–scattering was revealed for the first time in the measurements at 40 GeV/c [3] and considered as a new experimental regularity in the analyzing power t–dependence that time.

Oscillating pattern of analyzing power t-dependence with amplitude of oscillations increasing with t observed in various experiments in elastic and binary processes [4], is in conformity with the sum rule Eq. (2). Such oscillating dependence has obtained model explanation in [5]. It should be noted that the Eq. (1) does not imply similar $p_\perp$–dependence for the single–spin asymmetries in the inclusive processes and the corresponding experimental data have not revealed oscillations (cf. e.g. [1]).

Using rotational invariance combined with particle identity we can obtain similar sum rules for the spin correlation parameters in elastic and inclusive pp–scattering. Spin correlation parameters are the spin observables which describe dependence of the interaction on the relative orientations of the spins of the two particles (cf. [1]). We will consider scattering when both protons in *the initial state* are polarized. Definition of spin correlation parameter A_{nn} is the following

$$A_{nn}=\frac{\frac{d\sigma_{\uparrow\uparrow}}{dt}+\frac{d\sigma_{\downarrow\downarrow}}{dt}-\frac{d\sigma_{\uparrow\downarrow}}{dt}-\frac{d\sigma_{\downarrow\uparrow}}{dt}}{\frac{d\sigma_{\uparrow\uparrow}}{dt}+\frac{d\sigma_{\downarrow\downarrow}}{dt}+\frac{d\sigma_{\uparrow\downarrow}}{dt}+\frac{d\sigma_{\downarrow\uparrow}}{dt}}, \qquad (3)$$

where index n means that spins polarized along a normal to the scattering plane. Other parameters A_{ll}, A_{ss} and A_{sl} have the similar to Eq. (3) structure and differ by the orientation of spins in the initial state. Rotational invariance and particle identity leads to the following equality

$$\Delta\sigma_T^{el}(s)=-4\int_{-s+4m^2}^{0}A_{nn}(s,t)\frac{d\sigma}{dt}dt, \qquad (4)$$

where $\Delta\sigma_T^{el}(s)$ is the cross section difference with protons polarized along normal to beam direction:

$$\Delta\sigma_T^{el}(s)\equiv\sigma_{\uparrow\downarrow}^{el}(s)-\sigma_{\uparrow\uparrow}^{el}(s)$$

Parity conservation combined with particle identity allows us to get another relation

$$\Delta\sigma_L^{el}(s)=-4\int_{-s+4m^2}^{0}A_{ll}(s,t)\frac{d\sigma}{dt}dt, \qquad (5)$$

where $\Delta\sigma_L^{el}(s)$ is the cross section difference for the protons polarized along beam direction:

$$\Delta\sigma_L^{el}(s)\equiv\sigma_{\rightleftarrows}^{el}(s)-\sigma_{\rightrightarrows}^{el}(s)$$

We also have due to rotational invariance that

$$\int_{-s+4m^2}^{0} [A_{nn}(s,t) - A_{ss}(s,t)] \frac{d\sigma}{dt} dt = 0, \tag{6}$$

And parity conservation and rotational invariance provide

$$\int_{-s+4m^2}^{0} A_{sl}(s,t) \frac{d\sigma}{dt} dt = 0. \tag{7}$$

Similar relations can be written for the spin correlation parameters in the inclusive processes.

All above sum rules should be, of course, in agreement with the experimental data and therefore they can be used for the extrapolation to the region where data are absent at the moment. These sum rules are also interesting as a test ground for the models and must be obeyed under their construction.

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